# Effects of generative and discriminative learning on use of category variability

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**Abstract**

Models of category learning can take two different approaches to representing the relationship between objects and categories. The generative approach solves the categorization problem by building a probabilistic model of each category and using Bayes' rule to infer category labels. In contrast, the discriminative approach directly learns a mapping between inputs and category labels. With this distinction in mind, we revisit a previously studied categorization experiment that showed people are biased towards categorizing objects into a category with higher variability. Modelling results predict that generative learners should be more greatly affected by category variability than discriminative learners. We show that humans can be prompted to adopt either a generative or discriminative approach to learning the same input, resulting in the predicted effect on use of category variability.

**Keywords:** human category learning; generative models; discriminative models; rational models; Bayesian models

## Introduction

Categories can be learned using a variety of approaches. Here we examine two distinct approaches that humans can use to learn categories: *generative* and *discriminative*learning. While relatively unexplored in human categorization, this distinction has been widely studied in machine learning (e.g., Ng & Jordan, 2001). The distinction comes down to whether the ability to categorize objects is the result of estimating a distribution for each category, or learning a mapping from objects to categories. Both of these strategies can be used in learning real life categories. For example, you could learn the food preferences of a friend by observing the foods he eats and trying to infer a probability distribution, or by recording his affective responses to different kinds of foods and trying to identify which factors lead to positive or negative reactions.

More formally, generative and discriminative models represent two distinct strategies for estimating the probability that a particular object belongs to a category. Generative learners solve this problem by building a probabilistic model of each category, and then using Bayes' rule to identify which category was most likely to have generated the object. Discriminative learners estimate the probability distribution over category labels given objects directly. These different strategies have implications for the performance of these models. Theoretical and empirical analyses have shown that generative and discriminative models differ in their generalization behavior, as well as the speed and accuracy of learning (Efron, 1975; Ng & Jordan, 2001; Xue & Titterington, 2008).

While the generative/discriminative distinction has been studied extensively in machine learning and statistics, it has been little examined in human behavior. A recent study has shown humans can adopt these two different strategies while learning an artificial language (Hsu & Griffiths, 2009). In this paper, we explore whether people can adopt these two strategies in category learning.

The paper will be presented as follows. First we will provide an overview of generative and discriminative categorization models. Second, we will review related work from the existing human categorization literature. Third, we will revisit a previously studied paradigm that showed people are sensitive to category variability, being more likely to assign an object equidistant from the mean of two categories to the category with higher variance (Stewart & Chater, 2002; Cohen, Nosofsky, & Zaki, 2001; Rips, 1989; Smith & Sloman, 1994). Modelling results show that a generative model exhibits greater sensitivity to category variability than a discriminative model. We use this analysis as the basis for an empirical investigation of whether human learners can be prompted to take these two distinct learning approaches. Our results support the idea that humans adopt generative and discriminative approaches when appropriate. This provides new insight into the factors affecting human category learning.

## Generative and discriminative models

Rational models of categorization identify the underlying problem as one of estimating the probability of a given object $x$ belonging to a category $c$, as expressed by the distribution $p(c|x)$. The difference between generative and discriminative approaches to categorization comes down to how this probability distribution is estimated. Generative models build a probabilistic model of the input by learning the probability that an object $x$ is generated given that the category is $c$, $p(x|c)$, and then solving the categorization problem by applying Bayes' rule. Discriminative models estimate $p(c|x)$ directly. Generative models thus assume that observed objects are sampled in a way that reflects $p(x|c)$, while discriminative models do not make any assumptions about the distribution from which the input is sampled. These two approaches to categorization are illustrated schematically in Figure 1.

Comparison of generative and discriminative approaches to category learning has been done in the machine learning and statistics literature, where the classic *generative-discriminative pair* being compared is usually (generative) naïve Bayes vs. (discriminative) logistic regression (Efron, 1975; Ng & Jordan, 2001; Xue & Titterington, 2008). Under certain conditions, these two models are identical in the asymptotic form of the function $p(c|x)$ that they produce, differing only in how that function is estimated.

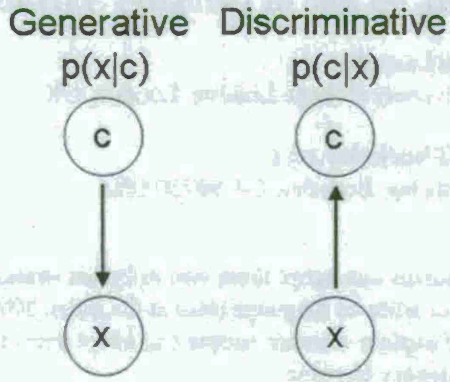


Figure 1: Generative and discriminative models. Generative models aim to estimate the probability distribution over the input given the category label. Discriminative models find a direct mapping between inputs and category labels.

Such generative-discriminative pairs can thus be used to explore the consequences of adopting these different strategies through mathematical analysis and simulations. For example, if the training data consist of two normally distributed samples, generative models learn categories more quickly (Efron, 1975; Ng & Jordan, 2001). However, when the training data come from other distributions, discriminative models are asymptotically more accurate (Xue & Titterington, 2008), though in some cases generative models may perform better initially and arrive at their (higher) asymptotic error more quickly (Ng & Jordan, 2001).

## Summary of related work

Previous models of categorization have used both generative and discriminative strategies, without necessarily recognizing that the significance of the distinction.The commonly cited prototype and exemplar models can be applied both generatively and discriminatively. Prototypes and exemplars are psychological models of category representation whereas discriminative and generative are statistical models of learning. Thus, prototype and exemplar models can be used under either approach, depending on how learning takes place. For instance, ALCOVE (Kruschke, 1992) is an exemplar model akin to discriminative kernel methods. SUSTAIN (Love, Medin, & Gureckis, 2004) is a discriminative model that chooses between exemplar and prototype representations. Decision bounds (Maddox & Ashby, 1993) can be either discriminative or generative depending on how model parameters are estimated. While rational models of categorization can adopt either approach, the ones proposed so far have taken a generative approach (e.g., J. R. Anderson, 1990; Griffiths, Canini, Sanborn, & Navarro, 2007). These generative categorization models span the range between exemplar and prototype representations. At the extremes, generative prototype models estimate parameters of category distributions (usually a Gaussian with a mean and variance) and generative exemplar models estimate category distributions using kernel density estimation (Ashby & Alfonso-Reese, 1995).

Despite the prevalence of human categorization models with both discriminative and generative approaches, most experimental paradigms seem more consistent with discriminative learning: stimuli are presented, participants guess the category and feedback is given. However, a few exceptions this can be seen in previous work on classification vs. inference learning, and observational vs. feedback learning. While not explicitly mentioned in previous work, both of these paradigms are potentially related to our discriminative vs. generative distinction.

### Classification vs. inference learning

Another line of experiments has shown that human category learning can also be influenced by using different tasks to teach people about the relationship between categories and features. The effect of using these two different tasks is similar to that of changing the direction of a learned causal relationship. (A. L. Anderson, Ross, & Chin-Parker, 2002; Markman & Ross, 2003; Ross & Murphy, 1996). In these experiments, all participants were presented with exactly the same training stimuli, consisting of the features and category membership of a set of objects. In one condition, learning took place via through *classification*: Participants were provided with the values for (some of) the features of an object asked to predict category membership. In the other condition, learning was based on making a predictive *inference*: The category membership and/or values of some of the features were provided and participants were asked to predict the value of another feature. Because participants in both conditions were given feedback, they were both ultimately provided with exactly the same information about categories and features. However, learning results differed in terms of performance accuracy and generalizations made. For example, inference learners performed better than classification learners on single-feature classification tasks but more poorly when all of the features were provided (A. L. Anderson et al., 2002). While this study was not motivated by generative and discriminative learning, people may have adopted these different strategies in the different conditions: Classification learning can be done using a discriminative model, while inference learning requires a generative model.

### Observation vs. feedback training

Another study, by Ashby, Maddox, and Bohill (2002), has also examined how learning of the exact same input was affected by presentation style. Here they compared what they called *feedback* training (where the category label appears after the object) with *observation* training (where the category label appears before the object). Their results showed that participants in the feedback condition performed significantly better than those in the observation condition for information-integration categories, where category membership could not be expressed in terms of a rule using a single feature. These two forms of training might encourage learners to adopt gen-

erative and discriminative strategies. Feedback training gives an error signal that can be used to adapt a discriminative model. Observation training is more relevant for learning object features based on the category label, which is the generative approach.

### Summary

Generative and discriminative models use different approaches to solve the problem of categorizing objects. Existing models of human category learning differ in which of these approaches they use. Previous work has not explored whether people are able to switch the approach they take in learning categories, although the effects of different training regimes that might encourage one approach over the other have been investigated. In the remainder of the paper, we explicitly test whether people can adopt these two approaches to learning categories, using a phenomenon that is diagnostic for one generative-discriminative pair of models.

## Differential use of category variability

Several experiments have shown an effect of category variability on human categorization judgments. In these experiments, the stimuli belong to one of two categories with different means and variances. The key question is how stimuli with features lying (perceptually) in between the two categories are categorized. The results of these experiments all showed that there was a bias towards categorizing stimuli into the high-variance category (Stewart & Chater, 2002; Cohen et al., 2001; Rips, 1989; Smith & Sloman, 1994). Here we propose that the degree of preference for the high variance category may be affected by whether the learner is adopting a generative or discriminative approach.

Intuitively, we expect category variability to have a greater effect on generative learners because estimating $p(x|c)$ for each category requires being sensitive to the variance of that category. In contrast, one need not consider the variance of the stimuli in simply learning a function from $x$ to $c$, $p(c|x)$. Indeed many discriminative models used in machine learning, such as support vector machines (Schölkopf & Smola, 2002), focus just on the location of the most extreme members of each category. We are not claiming that all generative models are sensitive to category variance, or that all discriminative models are insensitive, but that these approaches differ in the extent to which they are sensitive to this property of the stimuli. To illustrate this, we will explore the predictions of one generative-discriminative pair of models.

We follow previous work exploring the difference between generative and discriminative models (e.g., Ng & Jordan, 2001) and focus on the generative-discriminative pair of naïve Bayes and logistic regression. Since we will focus on continuous stimuli, we assume a Gaussian generative model, with

$$p(x|c = i) = N(\mu_i, \quad {}_i) \tag{1}$$

where $\mu_i$ and ${}_i$ are the mean and variance of the $i$th category with $i \in \{1,2\}$. The parameters $\mu_i$ and ${}_i$ can be estimated by maximizing the likelihood $\sum_{j=1}^{n} \log p(x_j|c_j,\mu, \quad)$, where $c_j$ and $x_j$ are the category membership and features of the $j$th stimulus respectively. The probability a novel stimulus belongs to a category, $p(c|x)$, is then computed by applying Bayes' rule, with the prior probability of each category being proportional to the number of observed stimuli from that category. The naïve Bayes model is similar to the Gaussian decision bound model used in Normal general recognition theory (Stewart & Chater, 2002; Maddox & Ashby, 1993).

The discriminative model uses logistic regression to estimate $p(c|x)$ directly, with

$$p(c = 1|x,w,b) = 1/(1 + \exp\{-w^T x) - b\}) \tag{2}$$

where $w$ and $b$ are the parameters of the model and $x$ is a vector of feature values. The parameters $w$ and $b$ are estimated by maximizing the log likelihood $\sum_{j=1}^{n} \log p(c_j|x_j,w,b)$. In general, $w$ and $b$ are vectors of length equal to the number of stimulus features. However, we will be using one-dimensional stimuli ($x_j$ is scalar), so $w$ and $b$ will be scalars in our case.

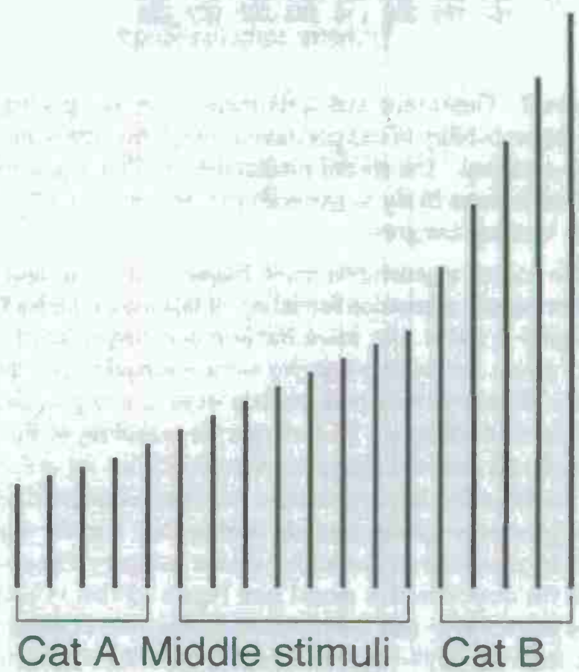


Figure 2: Stimuli used in the experiment. Category A and B were the low and high variance categories respectively

To examine the predictions of these models, we used stimuli based largely on those of Cohen et al. (2001). Stimuli consisted of vertical lines of varying lengths. Training stimuli belonged to one of two categories, A and B. Category A is the low variance category. Category A contained lines of length 110, 120, 130, 140 and 150 pixels. Category B was the high variance category. Category B contained lines of length 300, 375, 450, 525 and 600 pixels. All stimuli were equally likely within each category (categories had a flat distribution of stimuli). We also included novel transfer stimuli in the test

stimuli. There were eight transfer stimuli, equally spaced between the highest value of A and the lowest value of B (see Figure 2). A range of intermediate transfer stimuli were used in case the middle stimulus in psychological space differed from the numerical middle stimulus. The precise location of the middle stimulus is not important for our purposes, as the difference in results between generative and discriminative models is the question of interest.

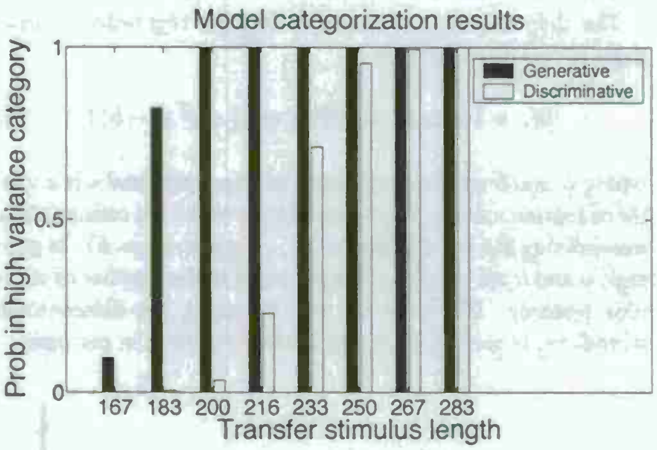


Figure 3: Generative and discriminative model predictions for the probability of categorization stimuli into the high variance category. The model predictions are that a generative learner is more likely to categorize in between stimuli in the high variance category

We trained a generative naïve Bayes model and discriminative logistic regression model on all labeled examples from category A and B. Our naïve Bayes model had uniform category priors, i.e. both categories were assumed to be equally likely. Parameters for both models were fit using maximum likelihood estimation. To compare the outcomes of the two models, we analysed categorization predictions for our transfer stimuli using these generative and discriminative models. The generative model predicts intermediate transfer stimuli will be classified to the high-variance category more often than the discriminative model (see Figure 3).This is because it is more likely that intermediate stimuli are extreme values from the high-variance category than the low-variance category. These results illustrate that sensitivity to category variability may be a diagnostic indicator of whether learners are using a generative or a discriminative strategy. In the next section we present an experiment that uses this indicator to determine whether human learners switch between these strategies depending on the way in which a categorization task is presented.

## Human generative and discriminative learning

### Method

**Participants** We collected data from 24 participants (12 in each condition). Participants wereundergraduates at the University of California, Berkeley and received course credit.

**Stimuli** Stimuli was the same training and transfer stimuli used in the model simulations described in the previous section. In the experiment, these stimuli were presented as white vertical lines in a black circle.

**Procedure** While previous related work had paradigms that may have encouraged discriminative or generative learning (Ashby et al., 2002; A. L. Anderson et al., 2002), the connection between these paradigms and the distinction was tentative. Thus, we will use our own experimental manipulation in order to encourage participants to adopt the distinct approaches as strongly as possible. Participants in both learning conditions were trained under the same randomized sequence of trials. In order to prompt generative or discriminative learning, the two conditions differed in the instructions, category-stimulus presentation order and question presented during testing blocks. Participants in both conditions were told they will see "signs" from an alien tribe. Participants in the *generative* condition were told that two aliens, one from each tribe (A and B) will appear and produce signs from their respective tribes. A picture of two aliens, who were identical except for the letter on their chest, was shown alongside the instructions. These instructions were intended to make it clear that the observed stimuli were generated from a probability distribution associated with the target category, consistent with the assumptions of a generative model. Participants in the *discriminative* condition were told that there are signs from two alien tribes and they would be shown a single alien translator who can report which tribe a sign was from. A single alien was shown alongside these instructions with a question mark on its chest. These instructions were intended to establish a situation in which participants learned a function from stimuli to category membership, consistent with a discriminative model.

For all participants, the experiment contained 10 blocks of 20 trials (each of 10 training stimuli were shown twice). Training blocks (odd blocks) were interleaved with testing blocks (even blocks). During training trials, participants were shown a black circular background on which the "sign" appears as a white vertical line, next to an alien with either A or B written on its chest. In the *generative* condition, the alien appeared 500 ms before the sign during training and the alien disappeared between trials to simulate different aliens appearing. In the *discriminative* condition, the sign appeared 500 ms before the alien and the alien did not disappear between trials to simulate one constant alien interpreter. In both conditions,once both stimulus and letter had appeared, both remained simultaneously on the screen for 1.5 s (see Figure 4). The total length of each training trial was 2 s and there were 700 ms between each trial.

During test trials, participants were shown a sign (white vertical line) on the black circular background. Participants in the *generative* condition were asked "Which alien was more likely to have produced this sign?". Participants in the *discriminative* condition were asked "Which alien tribe does this sign belong to?". Stimuli during each test block consisted of

every example stimulus in categories A and B, along with the eight transfer stimuli that were equally spaced between and highest value of category A and the lowest value of category B. (The highest value of category A and lowest value of category B were seen twice during each test block to make up the 20 trials.) No feedback was given during testing in either condition.

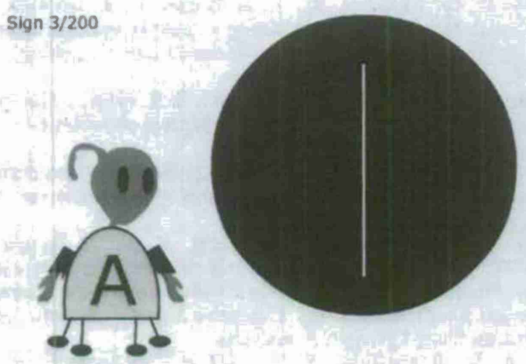


Figure 4: Screen shot of the experiment

## Results

The human learning results correspond to the predictions of the models: Generative learners are more likely to categorize transfer stimuli that lie in between the two categories in the high-variance category relative to discriminative learners (see Figure 5). A two-way within-between ANOVA revealed statistically significant effects of test stimulus ($F(9,198)$ = 76.88, $MSE$ = 0.036, $p < .001$) and condition ($F(1,22)$ = 5.43, $MSE$ = 0.216, $p < .05$) and a marginally significant interaction ($F(9,198)$ = 1.90, $MSE$ = 0.036, $p$ = .054). Planned comparisons using two-sample t-tests showed statistically significant effects of condition for stimuli 216 ($t(22)$ = 2.57, $p < .05$) and 233 ($t(22)$ = 2.46, $p < .05$). These statistics are calculated under the most conservative assumption, under which the responses from each participant for each stimulus are averaged together and treated as a single response.

The "middle stimulus" that lies midway between the two categories in human perceptual space (i.e. equally likely to be categorized in both categories in the discriminative condition) is of length around 200 pixels. This is smaller than the numerical middle (225 pixels). This is approximately the same value as the perceptual "middle stimulus" that was found in previous work (Cohen et al., 2001). Accounting for this shift, the discriminative model predictions match fairly well with the discriminative human results. The generative model predictions are significantly shifted to the left compared with our generative human results, meaning the generative model predicted an even stronger tendency to categorize the in-between stimuli in the high variance category. This difference in degree between model predictions and human judgments could be explained in many possible ways. One possibility is that perceptual stimuli might follow Weberian compression for the larger stimuli (Stewart & Chater, 2002). As a result of this compression, the perceptual variability of the longer length lines (which made up the high variability category) may have been significantly smaller than the absolute numerical variability values that were used in our models. If this were the case, a suitable transformation, such as to log space, would leave our qualitative results the same, while resulting in an appropriately less strong variability preference for the generative model. Another possibility is that people are not making the Gaussian assumption that was made by our model. This is plausible as our stimuli were very non-Gaussian. In this case, it is possible that the probability of belonging in the high variance category under a Gaussian assumption is greater than the probability estimates that generative participants might have made for our actual stimuli. Finally, participants may not be behaving fully generatively, or that the instructions resulted in a mixed population of generative and discriminative learners in this condition.

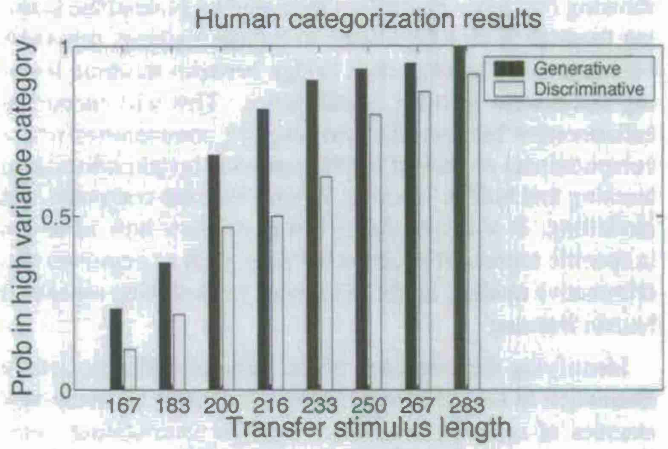


Figure 5: Probability of categorizing transfer stimuli in high variance category for participants in the generative and discriminative learning conditions. Total values are the average of all probabilities for individual stimulus lengths.

## Discussion

The distinction between generative and discriminative approaches to categorization has played an important role in machine learning research, but has not previously been explored in cognitive psychology. Our results show that people can be cued to take these two different approaches to category learning through the way in which a categorization task is presented. These results have implications for understanding human category learning, and for establishing links between the communities studying human and machine learning.

The finding that people behave differently when encouraged to adopt these two different approaches to category learning may shed light on previous empirical results in cognitive psychology. For example, some previous experiments have shown effects that may be partly due to learning paradigms that encouraged participants to adopt generative or discriminative learning approaches (e.g., Ashby et al.,

2002). The generative/discriminative distinction also has potential implications for previously proposed models of categorization. For example, it seems appropriate that connectionist models (Kruschke, 1992; Love et al., 2004) will best characterize behavior when humans adopt a discriminative learning approach whereas rational models (J. R. Anderson, 1990; Griffiths et al., 2007) will best describe behavior when humans adopt a generative learning approach. Developing a deeper understanding of how this distinction plays out in human learning may provide additional insights into long-standing debates on category learning.

Showing that people can adopt both generative and discriminative learning strategies establishes a new connection between human and machine learning. While many of the goals of machine learning are inspired by human capabilities (e.g., the ability to recognize and categorize complex structures quickly and efficiently), the principal issues that are topical in machine and human learning seldom coincide. By showing that a key distinction long studied in machine learning research is also significant to human learning, this work begins to build an important bridge between machine learning and human learning communities. This will encourage collaboration between the two research communities where computational models of learning provide insight into human learning and human learning, in turn, inspires computational modelling. It also establishes a way to know how advances in specific aspects of machine learning, such as improved discriminative models, might be relevant to predicting aspects of human learning.

Identifying the relevance of the generative/discriminative distinction in human categorization also opens up many new avenues of research questions. For the neuroscience community, one can ask: What neural mechanisms are implementing these two very different learning strategies? Are the neural circuits involved similar or different? This research also provokes many questions about learning more generally: When does human learning tend to be generative or discriminative? How flexible are learners in alternating between generative and discriminative learning approaches? Can learning approaches be retrospectively altered? (i.e. if input is learned with a discriminative perspective and learners were later made to understand that the data was generated from a probability distribution, would they switch their categorization judgments?) Since much of human learning in everyday life consists of a mix of scenarios in which one or the other of these strategies is more appropriate, clarifying when people use generative and discriminative approaches will help us understand differences in learning among individuals and across situations. We anticipate that exploring these questions will result in improved models of human category learning, and a tighter coupling between research on human and machine learning.

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To: AFRL/AFOSR Technical Reports
Cc: Prudhomme, Ian Lt USAF AFMC AFOSR/RSL
Subject: Final technical report for FA9550-07-1-0351
Attachments: SF298 - example.pdf

Please find attached the final technical report for AFOSR grant number FA9550-07-1-0351.

I have appended a list of publications resulting from this grant, together with links to electronic versions of these papers.

Please do not hesitate to contact me should you require further information regarding this project.

Best wishes,

Tom.

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Austerweil, J. L., & Griffiths, T. L. (accepted pending minor revisions). Seeking confirmation is rational for deterministic hypotheses. Cognitive Science.

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The proposed research had the goal of developing computational models of human inferences that bridge the gap between human and machine learning. This research focused on two components of cognition: learning about categories and their properties, and learning about causal and social relations. Research was completed successfully in both of these areas, resulting in a new unifying framework for models of category learning, new models for how people form and use high-level generalizations in causal learning, and new methods for predicting people's preferences and the relationships between them. The grant supported a total of 29 publications over three years, including one conference paper that won a best student paper prize, and provided support for five graduate students.

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